Practice Problems for Exam 1

Aside from having problems of the type below, the exam with also have true-false type questions and short answer questions that test concepts and definitions. Of course, there are many more problems below than will be on the exam – since these are practice problems.

Please note:

(i) **No calculators will be allowed during the exam.**
(ii) You must take the exam in class at our usual time. Attendance will be taken.
(iii) The exam will be posted on *Canvas*, and this is where you will upload the copy of your solutions.

1. Put the following matrices in reduced row echelon form and identify the rank of each matrix.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & -1 & 3 \\ -1 & 4 & 5 & -2 \\ 1 & 6 & 3 & 4 \end{bmatrix}, C = \begin{bmatrix} 3 & -2 & 1 & -2 \\ 1 & -1 & 3 & 5 \\ -1 & 1 & 1 & -1 \end{bmatrix}$$

2. Find the general solution (if it exists) for the system of equations corresponding to each of the following augmented matrices.

$$A = \left[\begin{array}{cccc|c} 1 & 0 & 3 & 3 & 1 \\ 0 & 1 & 6 & -1 & \sqrt{2} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right], B = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right], C = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

3. Find the solutions (if they exist) to the systems of equations below. If the system is homogeneous, find a basic set of solutions:

$$\begin{aligned} 3x - 3y + 15z + w &= 0 \\ 4x - 4y + 20z &= 0 \\ 3w &= 0 \\ 2x - 2y + 10z &= 0 \end{aligned}$$

$$\begin{aligned} x + z &= 1 \\ y + z &= 2 \\ x + y &= 3 \end{aligned}$$

$$\begin{aligned} 2x + 3y - 7z &= 2 \\ x + y - z &= 4 \\ 6x + 8y - 16z &= 10 \end{aligned}$$

4. Convert the system of equations to a matrix equation, and then solve the system by finding the inverse to the coefficient matrix.

$$\begin{aligned} x + y + 2z &= 5 \\ x + y + z &= 0 \\ x + 2y + 4z &= -2 \end{aligned}$$

5. For the matrices $A = \begin{bmatrix} 1 & 0 & -4 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & -1 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 9 \\ 0 & 1 \\ 3 & 3 \end{bmatrix}$, Calculate $4A^2 + 5B^t \cdot C^t$.

6. For the matrix $A = \begin{bmatrix} 2 & -4 \\ 6 & 8 \end{bmatrix}$, find A^{-1} in two ways: (i) using the formula involving the determinant of A and (ii) using elementary row operations with an augmented matrix. Use your answer in (ii) to write A^{-1} as a product of elementary matrices.